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Evaluation of Precise, Kinematic GPS Point Positioning

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BIOGRAPHIES

Dr. Oscar L. Colombo works on applications of space geodesy, and has developed and tested methods for highly accurate, very long baseline, kinematic GPS positioning. He has a degree in Telecommunications Engineering from the National University of La Plata, Argentina, and a Ph.D in Electrical Engineering from the University of New South Wales, Australia.

Andrew W. Sutter received a B.S. in Mathematics from Virginia Tech in 1997. He has been employed at the NSWCDD in Dahlgren, Virginia, since 1997 and has been involved in GPS testing since 1998.

Dr. Alan G. Evans has been working in Global Positioning System (GPS) applications at the Naval Surface Warfare Center, Dahlgren Division (NSWCDD) since 1981. He received the B.S.E.E. degree from Widener University in 1964, and the M.S. and Ph.D. degrees from Drexel University in 1967 and 1971, respectively.

ABSTRACT

We present the results of precise static and kinematic point-positioning solutions obtained in post-processing, with a recursive Kalman filter and smoother estimator (i.e., a state space approach). The observations are dual-frequency GPS carrier phase and pseudo-range, treated as two distinct data types, each with its own measurement equations. The receiver data is single-differenced between satellites, to eliminate the receiver clock, and processed using precise satellite ephemerides and clock corrections available from the Analysis Centers of the International GPS Service (IGS), while estimating: receiver coordinates, ionosphere-free carrier phase Lc biases, and tropospheric refraction model errors. Phase windup, relativity, satellite and receiver antenna corrections are all applied, as well as receiver position corrections for earth tides, ocean loading, and pole tide, in accordance with the IERS Conventions 2003. These features are part of a general upgrade to long-baseline differential software created by the first author, which now can be used also in point-positioning, and in mixed modes. Part of the motivation has been to gain experience that could be useful in developing a real-time system. The point-positioning accuracy has been evaluated using data from different test data sets, including fixed receivers at known locations and a moving receiver mounted on a van. The receiver positions so obtained have been verified using as "truth", in some tests, the known coordinates of the fixed sites, and, in the case of the van, a precise trajectory, differentially determined relative to a nearby receiver, with correctly fixed carrier phase ambiguities. The results indicate a precision at the level of a few centimeters, for static solutions, and below one decimeter, in kinematic mode.

INTRODUCTION

Precise point positioning is the geolocation of a fixed or moving object, using data from a single GPS receiver, and precise satellite ephemerides and clock corrections.

Work on very precise static point positioning, in the middle to late 1990's [1], [2], [3] indicated that, at least for static solutions, point-positioning solutions could be as exact as differential ones (i.e., of centimeter-level precision). At the time, the use of precise point positioning was limited, for most users, to static applications, because GPS data could be used only if collected at the exact same epochs for which there were high-quality clock corrections. (Typically, these were, then, available at five-minute intervals, from IGS data archives, for use only in post-processing.) This was so because the satellite clocks were dithered with a pseudo-random signal that made clock errors unpredictable at other epochs, for most users. This was known as Selected Availability, or SA, and was intended to limit the access to precise real-time GPS navigation only to approved users. In early May of 2000, the US Government changed its policy, and SA was turned down to its lowest possible level. Since then, it has become possible for all users to interpolate accurately the clock estimates, and correct data collected at much higher rates. This has opened the way for, potentially, vast numbers of users everywhere, to find very accurately, each with a single receiver, not just the positions of fixed sites, but also the trajectories of all kinds of vehicles. The practical implications are great, and they are still being studied and understood, as experience accumulates, and new ideas are tested. One very important development is the attempt to make real-time point positioning available worldwide [4].

The ability to use GPS to find accurately the position of a vehicle without the need for a reference base station, opens the way to many interesting forms of wide-area remote sensing in real time. Possible applications include: photogrammetry, mapping terrain with scanning radar, lidar, or sonar, the advance warning of natural hazards such as tsunami, deep-ocean hydrographic surveys, etc.

For the present work, software originally developed by the first author for precise differential positioning has been upgraded, and a point-positioning option, added. As in the original version, the solutions are made in post-processing, or off-line, that is, after the fact. However, part of the motivation for the upgrade has been to gain experience with point positioning, so as to include, eventually, a similar feature in a real-time system now being developed [5] at the US Navy's Naval Surface Warfare Center, Dahlgren Division (NSWCDD), in Dahlgren, Virginia.

TECHNIQUE

Both the original procedure [6], [7], and the corresponding software, were designed for doing precise differential kinematic and static positioning over very long baselines, although including features commonly found in short-baseline procedures, such as on-the-fly ambiguity resolution, and stop-and-go and rapid-static (or pseudo-kinematic) solutions. In the updated version, as in the original, the data are dual-frequency carrier phase and pseudo-range observations, treated as two distinct data types, each with its own set of measurement equations. The post-processing estimator is recursive, consisting of both a Kalman filter and a smoother (state-space approach). The smoother is only needed for kinematic solutions

To include a point-positioning option, a number of modifications were made. First, several new corrections to the data were added: for precise satellite clocks, general relativity, phase windup (along with a refinement of the models for Block II satellites' orientation), better tropospheric refraction[1] and earth-tide models, and ocean loading and pole tide corrections. Further, the Nuvel plate tectonics model for reducing site coordinates to a common epoch if no site velocities are available, and receiver antenna-pattern corrections have been added, as options, for use in precise static solutions. Moreover, for a different project [8], the ability to adjust GPS orbits in differential mode, so as to obtain precise results with broadcast orbits, is being extended to include the refinement of the trajectories of Low Earth Orbit (LEO's) satellites.

The point-positioning solutions are based on GPS data single-differenced between satellites, to eliminate the receiver clocks. To introduce this form of point positioning with a minimum of modifications to the original differential procedure, which is based on the use of double-differences, a *fictitious* base station has been introduced, with the same coordinates as the actual single receiver. But, instead of GPS observations, this “station” data is actually the sum of the clocks and relativity corrections for the rover data. Both the corrections and the data are single-differenced between each satellite observed by the rover, and one particular satellite, chosen to be the common interferometric reference. Subtracting the single-differenced values of the fictitious station from the corresponding single-differenced data of the rover, “double-differences” are formed that are, in fact, the single-differences between satellites of the rover's data, corrected for clocks and relativity. These are then processed, formally, as if they were double differences, with only a modicum of additional changes to the procedure. An important consequence of this is that the software can still be used in differential mode. In fact, a mixed mode is also available, where the between-satellites single differences of the rover can be combined with true double differences relative to fixed base-station receivers.

Precise clock orbits and corrections are available online from archives such as the one kept by the Crustal Dynamics Data Information System (CDDIS) at the NASA's Goddard Space Flight Center. They are in computer files deposited there by various IGS contributing Analysis Centers. The satellite positions are given once every fifteen minutes, and the clocks are given mostly at 5-minutes' intervals, although both JPL and the University of Bern contribute them also at 30-seconds' intervals. These clock corrections are linearly interpolated to the epochs of the data, usually collected at a higher rate than that of the clocks. The orbits are interpolated with a 15^{th} order Lagrange interpolator. Because the interpolated clocks and orbits, however precise, are never perfect, their combined error at each epoch is taken into account by either adding extra measurement noise, or by solving for clock errors, treated as white noise processes. The latter can be done in static point-positioning solutions, and mix-mode solutions.

SOFTWARE

The post-processing software “IT” (“Interferometric Translocation”) has been developed by the first author, over a period of several years, as part of a series of

[1] Niell’s mapping functions. Meteorological data, optional, with Saastamoinen’s wet and dry zenith delay models.

collaborations with university and government groups in the USA and abroad. A particularly fruitful one has been the ongoing effort, in which the three authors participate, to develop precise kinematic techniques at NSWCDD.

The "IT" software, written in Fortran, has been compiled and used successfully in a number of operating systems. These are listed in the overview of the software given in Table 1. Also a port to the Macintosh System X is now in the works.

There are two main programs: the first one ("prp") is used to pre-process the data. This means to edit out bad observations, apply the satellite clock corrections and the general relativity correction, repair carrier-phase cycle-slips, and form single (or double) differences. The other program ("ngps") is used to analyze the pre-processed data, and make the precise static or kinematic solutions.

To process the data for the examples given in this paper, less than five minutes of computer time were needed in each case, using an older 266 MHz Pentium II laptop computer, with 64 Mb of RAM, and Windows 98. With newer machines, that time can be considerably shorter.

Features Inherited from the Differential Version. All capabilities implemented in the original version are still available in differential mode, and most of them, also, in point-positioning mode. Among these, for example, is the ability to estimate and correct errors in the GPS satellite orbits most immediately accessible to all users, those broadcast in the Navigation Message. This is an option in differential and mixed modes only (since simultaneous data from more than one receiver are needed). Other aspects, still present in the new version, are explained below, and more are outlined in Table 1.

The Kalman filter is updated every few minutes, with compressed data (or "normal points"), to reduce drastically the number of updates, the computing time, the amount of data passed from filter to smoother in post-processing, and the arithmetic round-off error propagation. In kinematic mode, estimated unknowns are then used to correct the full-rate data, to obtain the instantaneous rover positions.
Besides vehicle coordinates, refraction, and the biases of the ionosphere-free carrier phase combination, or Lc, the unknowns solved may include also: orbit errors (initial position, velocity, and unmodeled accelerations), and reference station coordinate errors. The orbit error treatment is based on analytical orbit theory [8], [9]. Both carrier-phase and pseudo-range are processed in the navigation Kalman filter using complete observation equations. The observation equations, at each epoch, are linearized about a nominal position of the vehicle determined from pseudo-range data only, with *a priori* standard deviations of 100 m per coordinate, and "white noise" or "zero-memory" dynamics (i.e., a purely kinematic treatment). The *a priori* values of the carrier-phase Lc biases are the differences of instantaneous, double-differenced phase and pseudo-range observations. Each bias has a 10 m *a priori* standard deviation, and is treated as a constant (assuming all cycle-slips have been properly corrected by the real-time pre-processing subroutines). For estimating broadcast orbit errors--in differential mode--the *a priori* value for each initial position and velocity component is zero, with a standard deviation (STD) of 4 m, and 0.1 mm/s, respectively. Satellite force model errors are treated as unknown, piece-wise random-walk accelerations in the radial, along, and across-track directions ("reduced dynamics" approach), with STDs of 10^{-9} m/s^2, that change value every few minutes. Satellite clocks are treated as white noise. Each zenith delay error state is a random walk with an initial *a priori* value and STD of zero and 0.1m, respectively, driven by ~1 cm/(hour)$^{1/2}$ STD of process noise. North and East troposphere gradients are also treated as random walks with ~0.1 mm/(hour)$^{1/2}$ STD of process noise. (The user can change most STDs in the job setup.)

KALMAN FILTER CONVERGENCE

To obtain a precise solution, enough data has to be assimilated by the navigation Kalman filter to estimate well the nuisance unknowns. In point positioning, these are the Lc biases and the residual refraction zenith delays. Only then can the receiver position be found with high precision. Of those nuisance unknowns, the Lc biases are the most influential in determining the quality of the results. It is important to get them estimated quickly, so the solution converges to its full precision as soon as possible. Convergence time matters, not only in real time applications, but also in post processing, particularly when the observing session is short, or there are many breaks in the data that require restarting the filter often. It is a particularly important issue in kinematic solutions, where convergence is usually much slower than in static ones. One way of speeding up convergence could be to resolve, when possible, the L1 and L2 ambiguities, which can then be combined to get the Lc biases
The "IT" software has three modes of operation: differential, point positioning, and mixed. In differential and mixed modes there is the option, in addition to solving for Lc biases, to resolve ambiguities on the fly over short baselines (less than 20 km), and over long baselines (up to 600 km), if a good ionospheric correction is available [10]. As with all ambiguity resolution methods, the geometry-free one implemented in "IT" takes advantage of the integer nature of the double-difference ambiguities. However, carrier phase ambiguities, in point positioning, have fractional numbers of cycles, because of unknown transmission delays in the satellites. (In differential positioning, the fractional parts can be eliminated by forming double differences.) The hardware delays can be calibrated, in principle, using observations from ground

receivers at fixed, known locations. If so, the fractional part of the ambiguities in the single-differenced phase data can be measured and corrected out in the same way as the clock errors, leaving only the integer part to be resolved. A very good ionospheric correction will be needed, as well, in order to resolve those integers. However, at this time, the authors are not aware of any practical method for doing this. In such a fast-developing field, it seems reasonable to hope that such a method might be found, eventually.

Another way to speed up convergence is by using both carrier phase and pseudo-range data. One simple way to do this would be to subtract the pseudo-range from the Lc phase, forming a time series of noisy estimates of the biases (assuming all cycle-slips have been detected and corrected), which then can be averaged together, to reduce their noise. A more effective approach is to use both types of data to update the navigation Kalman filter where the Lc biases are solved as real-valued unknowns (floated). One problem using pseudo-range data, particularly with older types of receiver, could be the presence of strong low-frequency multipath, which is hard to filter out quickly. Finally, in some situations it is possible to introduce special constraints in a solution, to speed up convergence. One example is the *mean sea level variability constraint* [11], which can be used with ships and buoys floating at sea, in lakes, etc. Another example is the use of precisely pre-positioned control points, as explained later, when discussing one of the tests.

TABLE I

MAIN FEATURES OF THE PRECISE POSITIONING SOFTWARE "IT"

<u>Operating Systems</u>:

UNIX, LINUX, FreeBSD, WINDOWS 98, ME, NT, 2000, XP.

<u>Data</u>:

GPS dual-frequency carrier phase and pseudo-range, treated as separate data types. Data Rates: 0.03Hz-10Hz. (Different reference and rover receiver rates allowed in differential mode, without SA)

<u>Recursive Estimator</u>:

Kalman Filter and Smoother,
with Data Compression (Normal Points)

<u>Solution Types</u>:

Static
Kinematic
Surveying (Stop-And-Go, Rapid Static)
Reduced Dynamics: GPS satellites (LEOs, experimental)

TABLE I (Contd.)

<u>Basic Modes</u>:

Differential (Multiple Baselines)
Point-Positioning
Mixed

<u>Data Corrected for</u>:

Satellite Clocks, General Relativity, GPS Antenna Offset, Phase Windup, Light Flight-Time, Tropospheric Refraction (Niell, Met. data optional), Receiver Antenna Offset and Pattern, Earth Tides, Site Velocities (Default: NUVEL tectonics model), Ocean Loading, Pole Tide.
(Compliant with the International Earth Rotation Service Conventions 2003)

<u>Unknowns</u>:

Site/Vehicle Coordinates,
Lc Biases (L1, L2 integer cycle ambiguities resolved, in differential and mixed modes, when possible.)
GPS Orbit Errors (differential and mixed modes), Clocks, Residual Zenith Delays, E and N Horizontal Troposphere Gradients (in static mode).

EXAMPLE No. 1: STATIC SOLUTIONS

Static solutions are, in general, more precise than kinematic ones and, therefore, better for finding small systematic errors that may go otherwise undetected. Precise static solutions are also, of course, of great practical value in surveying, and of scientific value in geophysics.

Figure 1, below, shows the world-wide distribution of ten IGS sites used to test the point-positioning method in static mode, as implemented in the "IT" software. The solutions were all for 24-hour sessions spanning the same day (4 May 2002). The 15-minute precise orbits and 5-minute clocks for that day, and the corresponding site coordinates, were all from the same IGS solution for that week. Since the data used in our solution were also used, most likely, in the IGS solution (a weighted combination of solutions from several contributing Analysis Centers), one should expect our positions to be in good agreement with the IGS ones, if our software worked correctly. The only obvious difference in procedure is that ours are 24-hour solutions, while that of the IGS is a 7-day one. So, if the point-positioning technique has been implemented correctly, the agreement, while not exact, should be very close. Figure 2 shows the differences in height, and in the northern and eastern directions, between the IGS coordinates and ours, for all ten sites. (This comparison is conservative, since we did not make a 7-parameter transformation to eliminate any possible difference in reference frames.) All discrepancies are of less than 2 cm in absolute value. Interestingly, this is the same level of agreement achieved, nowadays, with the long-baseline differential approach.

Note on the Use of Precise Orbits and Clock Corrections. Clocks from the IGS or other sources must always be used with orbits from the same source and, in particular, from the same global solution. This is so because neither the precise orbits, nor the clock corrections are perfect: they may have significant, but highly correlated errors. These will largely cancel each other out, when used together, if the orbits and clocks have been estimated together.

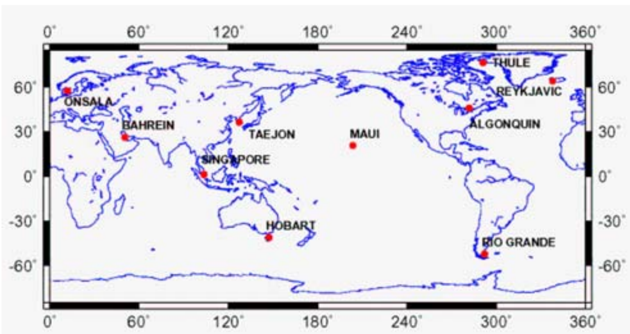

Figure 1. Map showing the locations of the ten IGS sites for which 24-hour static solutions have been made, and later compared to their values from the IGS solution for the same week (the day was May 4, 2002).

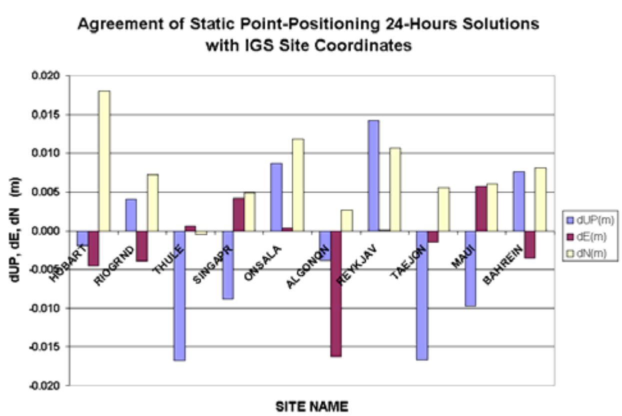

Figure 2. Bar chart showing the differences in Height, North, and East (in meters), between each of the ten static solutions and the corresponding IGS site positions.

From our experience after a number of different tests, including those reported here, more frequent clock corrections make for better results. At present, clocks are available at 5-minute intervals, from most IGS contributors, and every 30 seconds from NASA's JPL, and from the University of Bern. In general, we have noticed a 30% improvement in the consistency of point positioning results with "truth", when using the more frequent 30-second corrections.

EXAMPLE No. 2: KINEMATIC SOLUTIONS AT FIXED SITES

Figure 3 shows the comparison of the instantaneous kinematic positions of the IGS site in Thule, Greenland, with the precise IGS coordinates for that site (with data from that site collected on the 25th of May of 2004). In this case, we used 30-second clocks and corresponding orbits from CODE, at the University of Bern.

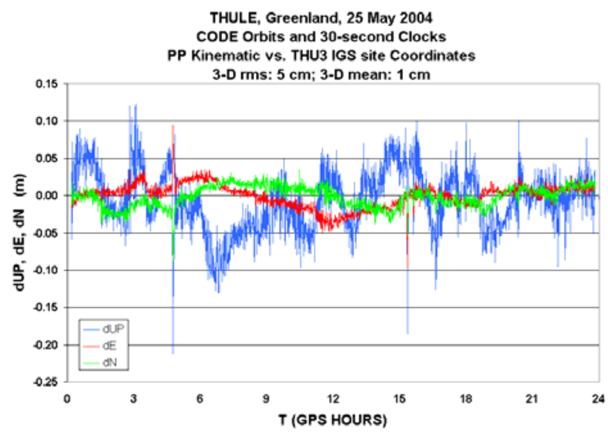

Figure 3. Comparing a 24-hours point-positioning kinematic solution with the precise coordinates for the IGS site at Thule, in Greenland (y-axis values, in meters).

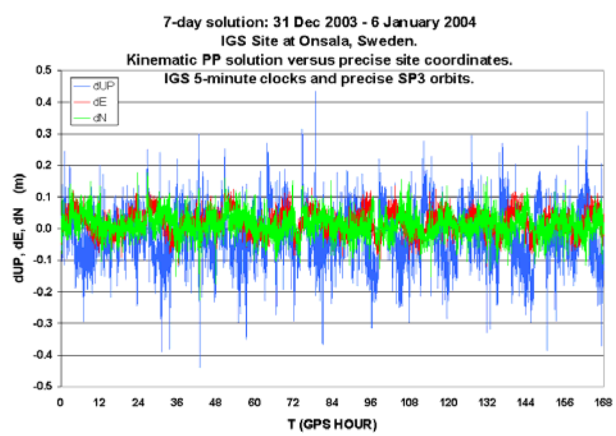

Figure 4. Comparing a 7-days point-positioning, kinematic solution, to the precise IGS coordinates for the IGS site at Onsala, in Sweden (y-axis values, in meters).

As in the static solutions before, the earth tide movement at that site has been corrected, in order to compare our results to precise, fixed, "tide-free" site coordinates. Figure 4 shows a similar comparison using the IGS site at Onsala, Sweden, in this case for a seven-day point-positioning, kinematic solution. In both cases, the GPS data were available from the CDDIS archive at intervals of thirty seconds. For Onsala, the orbits and clocks were from the IGS combined solution for that week, with 5-minute clock corrections.

Summary Statistics. For the kinematic solutions for Thule and Onsala, the discrepancy with the known coordinates of those sites can be expressed in terms of their 3-D RMS and their 3-D Mean. The former is the square root of the sum the squared RMS values of the Height, North, and East discrepancies, while the latter is the modulus of the 3-D vector with the Height, North, and East mean discrepancies for components. For Thule, these numbers are: the 3-D RMS, 4.6 cm, the 3-D Mean, 0.7 cm. For Onsala, the 3-D RMS is 9.3 cm, the 3-D Mean, 3.1 cm.

EXAMPLE No. 3: THE VAN TEST AT NSWCDD

On the 27th of April of 2004, there was a test of a real-time kinematic system, during which a van was driven inside NSWC, Dahlgren Division, for about one hour, first in the morning and, again, in the afternoon. Some of the data collected that day were meant to be used, also, to test the off-line, point-positioning technique discussed here.

In addition to two GPS receivers in the van (sharing one antenna), there was another receiver at a fixed, precisely known location (BARN). The fixed site and the vehicle were never more than 1.5 km from each other. So it was possible to resolve the phase ambiguities in L1 and L2, obtaining, for the morning and the afternoon sessions, very precise, L1-only, differential, short baseline solutions for one of the van receivers (TRK2). The resulting trajectories were used as the "truth" to which the post-processed point positioning solutions were compared. Both the fixed and the moving receivers were Ashtech Z-12s, collecting data at 1-second intervals.

The precise orbits and clocks were from CODE, University of Bern. From early April this year (2004), this group started making available their high-quality 30-second clock solutions, through the CDDIS online archive.

Figure 5 shows the route followed by the van during the morning session, much the same as that along which it was driven in the afternoon. Figures 6 a, and b, show the comparison between "truth" and the filter-smoother solutions for both morning and afternoon. Clearly, their agreement in the afternoon was better. To understand why, one must turn to Figures 7 a, and b. They show the trajectory estimated with the filter only, compared to "truth". The increasing agreement, as the filter converges as more and more data are assimilated, is clearly seen. In the case of the afternoon session, the filter converges to a fairly high level of precision during the last few minutes. In the morning, the filter does not have time to converge that much. Since the precision throughout a smoothed solution is roughly that of the filter solution at the end of the session, the morning results must look worse than those of the afternoon.

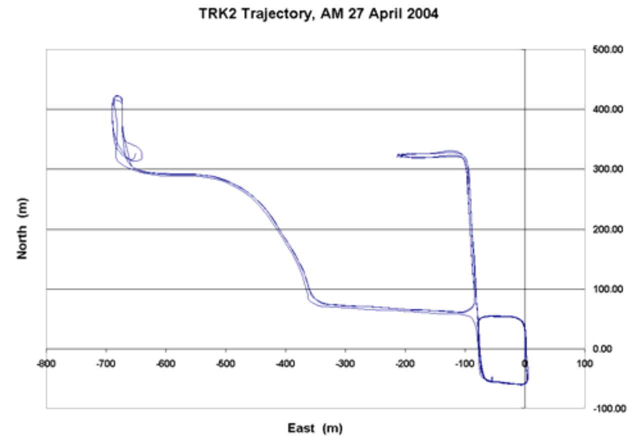

Figure 5. Trajectory of the NSWCDD van, morning run, 27 April 2004.

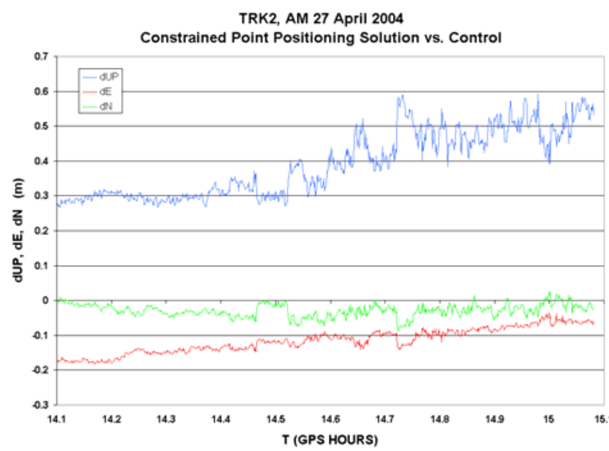

Figure 6 (a). Agreement of the post-processed, point-positioning solution, with the precise "truth" trajectory, morning session.

This situation, where in only one of two roughly equally long sessions there was enough time to achieve high precision, illustrates the problem of slow filter convergence, particularly when sessions are short. Of those remedies suggested in the section on Filter Convergence, the pseudo-range data have been used in combination with the carrier-phase in both solutions, though unsuccessfully. But there is another approach, explained next, which, when tried, produced the improved results shown in Figure 8. (Plots 7 (a), (b) look smoother than the others, because the filter is updated only once every two minutes.)

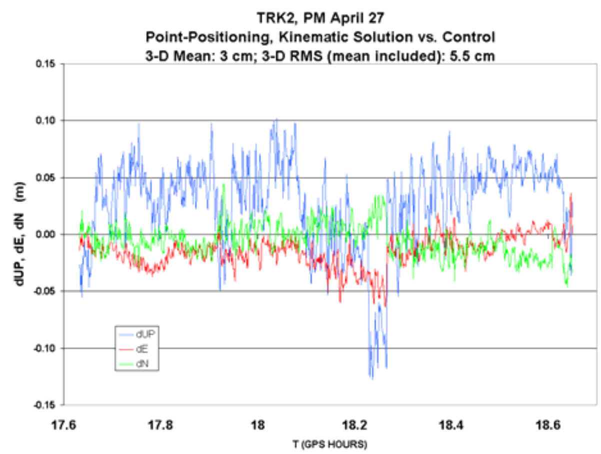

Figure 6 (b). Agreement of the post-processed, point-positioning solution, with the precise "truth" trajectory, afternoon session.

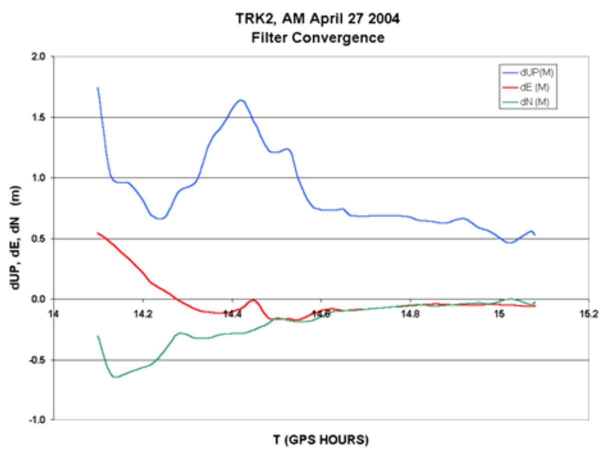

Figure 7 (a). Convergence of Kalman filter solution, morning session. (Filter updated every two minutes.)

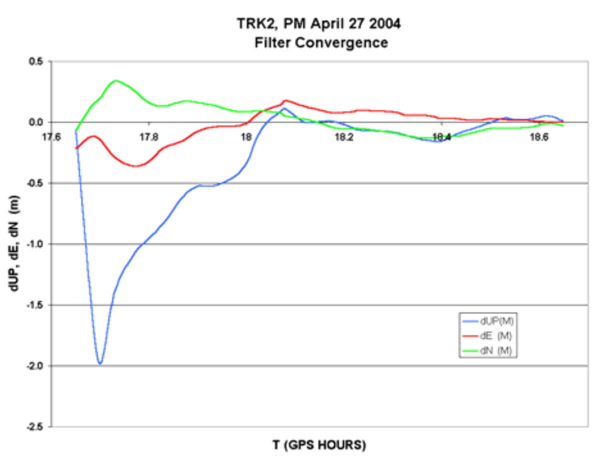

Figure 7 (b). Convergence of Kalman filter solution, afternoon session. (Filter updated every two minutes.)

During the first two minutes, while the vehicle was stationary, the initial points of the trajectory were constrained to be the same as those in the precise "truth" solution. This was enough to make the solution converge immediately. In the later part of the session, when some satellites were just rising while others were setting, new and, as yet, unknown Lc biases appeared, and already well-determined biases disappeared, so there was a gradual loss of precision. Repeating the constraining when the van was stationary on another known location, could have brought the agreement with "truth" back to its initial level of precision. This could not be done, however, because the receiver on the van was turned off before the van stopped.

Summary Statistics. The fit of the point-positioning trajectories to the control, or "truth", solutions for the van test can be summed up with the following statistics: For the constrained morning solution, the 3-D RMS is 7.7 cm, the 3-D Mean, 6 cm. For the afternoon solution, the statistics are: 3-D RMS, 5.5 cm, and the 3-D Mean, 3 cm.

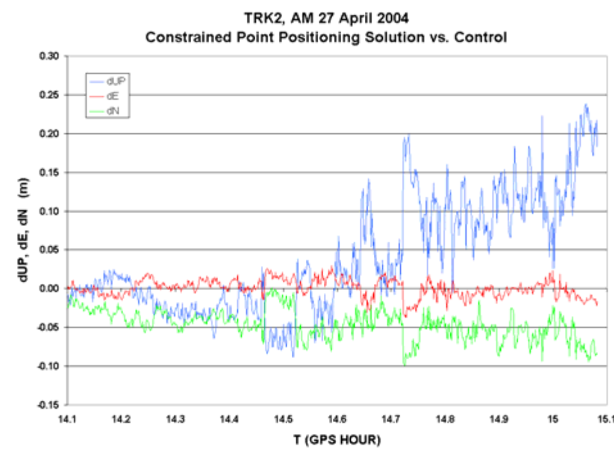

Figure 8. Post-processed solution with an initial position constraint, morning session.

These observations have practical implications. Imagine a group of archaeologists, geologists, or engineers, surveying the site of a dig, a future mining operation, or a future road, bridge, or dam. They are in some inaccessible and remote region, for example, in the middle of a desert, or of some rugged mountains. The first day, while setting up camp, they keep a GPS receiver operating for the whole day at a convenient location, in order to set up one initial control point for their subsequent surveys. For this, they post-process the data that evening, in point positioning mode, using (say) IGS "rapid" orbits and clock corrections obtained through a radio connection to the Internet. The next morning, at the start of their first kinematic survey, they stop for a few minutes at that control point, and then proceed to other convenient points chosen throughout the area, which they occupy also for a few minutes. A subsequent post-processed, point-positioning, stop-and-go solution, using the previously determined location of the starting point as a constraint, will give them the precise positions of those other points. Which, in this way, become new control points, conveniently distributed over the whole area. Now they are in a position to continue to survey quickly and with nearly the same precision as that of the nearest control point. The same idea can be used in differential mode.

CONCLUSIONS

Based on our experience with both static and kinematic point positioning, we observe that:

(1) The three-dimensional RMS and mean of the discrepancies between the point-positioning solutions and the precise, independent solutions used as "truth", are of the order of a few centimeters in the static case, and are less than 10 cm in the kinematic case. This agreement is much the same as previously recorded, by us and by others, in similar tests of long-baseline differential positioning.

(2) The degree of convergence of the Kalman filter at the end of a session, or just before a prolonged data break that necessitates restarting the filter, largely determines the precision of the whole post-processed (i.e., filtered and smoothed) solution. This convergence tends to be slow, particularly in kinematic mode: typically 30-40 minutes, sometimes even longer. But there are ways of speeding it up, in some cases, such as the use of low-multipath pseudo-range, a dynamic constraint on the change in mean height on ships or buoys, or the occupation of marks of known coordinates in stop-and-go surveys. Slow filter convergence is an important issue, particularly in real-time applications of point positioning, and less so in post-processing, except with short sessions.

(3) How often the precise satellite clock corrections are available can make a significant difference: 30-second clocks are preferable to 5-minute clocks, at least when interpolating linearly to higher data rates. Clock interpolation deserves further study.

ACKNOWLEDGEMENTS

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